

Question	Scheme	Marks	AOs
1	Finds critical values $x^2 - x > 20 \Rightarrow x^2 - x - 20 > 0 \Rightarrow x = (5, -4)$	M1	1.1b
	Chooses outside region for their values Eg. $x > 5, x < -4$	M1	1.1b
	Presents solution in set notation $\{x : x < -4\} \cup \{x : x > 5\}$ oe	A1	2.5
		(3)	
<b>(3 marks)</b>			
<b>Notes</b>			
<p><b>M1:</b> Attempts to find the critical values using an algebraic method. Condone slips but an allowable method should be used and two critical values should be found</p> <p><b>M1:</b> Chooses the outside region for their critical values. This may appear in incorrect inequalities such as <math>5 &lt; x &lt; -4</math></p> <p><b>A1:</b> Presents in set notation as required <math>\{x : x &lt; -4\} \cup \{x : x &gt; 5\}</math> Accept <math>\{x &lt; -4 \cup x &gt; 5\}</math>. Do not accept <math>\{x &lt; -4, x &gt; 5\}</math></p> <p>Note: If there is a contradiction of their solution on different lines of working do not penalise intermediate working and mark what appears to be their final answer.</p>			

Question	Scheme	Marks	AOs
2	Complete method to find the RHS of an equation for $l$ e.g., Attempts gradient = $\frac{80-60}{10} \{=2\}$ and uses intercept = 60	M1	1.1b
	$\{y=\}2x+60$	A1	1.1b
	Deduces the RHS of the equation for $C$ is $\{y=\}ax(x-6)$ and attempts to use (10,80) to find the value of $a$	M1	3.1a
	Equation of $C$ is $\{y=\}2x(x-6)$	A1	1.1b
	$2x(x-6) \leq y \leq 2x+60$	B1ft	2.5
		(5)	

(5 marks)

**Notes:**

**M1:** Complete attempt to use the given information to find an equation or inequality for  $l$ , which may be  $l =$  or have no LHS.  $y - 80 = 2(x - 10)$  is acceptable for this mark.

**A1:**  $\{y=\}2x+60$  This is not scored by  $y - 80 = 2(x - 10)$

**M1:** Deduces the RHS of the equation of  $C$  is  $\{y=\}ax(x-6)$ ,  $a \neq 1$ , and attempts to use (10,80) to find the value of  $a$  which may be implied. Again, there may be no LHS.

Other possible and more lengthy alternatives include:

1) Setting the RHS to be  $\{y=\}a(x-3)^2 + b$  and using (0,0) and (10,80) to find  $a$  and  $b$

2) Setting the RHS to be  $\{y=\}px^2 + qx$  and using (6,0) and (10,80) to find  $p$  and  $q$

**A1:**  $\{y=\}2x(x-6)$  or alternative such as  $\{y=\}2(x-3)^2 - 18$  or  $\{y=\}2x^2 - 12x$

This may be implied by an inequality  $y \dots 2x(x-6)$  and may be seen as, e.g.,  $C = 2x(x-6)$

**B1ft:** " $2x(x-6) \leq y \leq 2x+60$ " o.e. must follow from their  $l$  and  $C$  and apply isw

Follow through only on a quadratic for  $C$  and a straight line for  $l$

Do not allow a mixture of inequalities, i.e.,  $<$  with  $\leq$

Allow  $2x^2 - 12x < y < 2x+60$  or as separate inequalities  $y > 2x(x-6)$ ,  $y < 2x+60$

Do not allow  $2x(x-6) < R < 2x+60$  or  $2x(x-6) < f(x) < 2x+60$  or  $2x(x-6) < 2x+60$

Ignore any reference to  $-3 < x < 10$

Note:  $y = 2x+60$  and  $y = 2x(x-6)$  incorrectly expanded to  $y = 2x^2 - 12$  followed by

$2x^2 - 12 \leq y \leq 2x+60$  would score 11110

Question	Scheme	Marks	AOs
<b>3 (a)</b>	$2 < x < 6$	B1	1.1b
		(1)	
<b>(b)</b>	States either $k > 8$ or $k < 0$	M1	3.1a
	States e.g. $\{k : k > 8\} \cup \{k : k < 0\}$	A1	2.5
		(2)	
<b>(c)</b>	<b>Please see notes for alternatives</b>		
	States $y = ax(x-6)^2$ or $f(x) = ax(x-6)^2$	M1	1.1b
	Substitutes (2,8) into $y = ax(x-6)^2$ and attempts to find $a$	dM1	3.1a
	$y = \frac{1}{4}x(x-6)^2$ or $f(x) = \frac{1}{4}x(x-6)^2$ o.e	A1	2.1
		(3)	
<b>(6 marks)</b>			
<b>Notes: Watch for answers written by the question. If they are beside the question and in the answer space, the one in the answer space takes precedence</b>			

(a)

B1: Deduces  $2 < x < 6$  o.e. such as  $x > 2, x < 6$   $x > 2$  and  $x < 6$   $\{x : x > 2\} \cap \{x : x < 6\}$   $x \in (2, 6)$

Condone attempts in which set notation is incorrectly attempted but correct values can be seen or implied E.g.  $\{x > 2\} \cap \{x < 6\}$   $\{x > 2, x < 6\}$ . Allow just the open interval  $(2, 6)$

Do not allow for incorrect inequalities such as e.g.  $x > 2$  or  $x < 6$ ,  $\{x : x > 2\} \cup \{x : x < 6\}$   $x \in [2, 6]$

(b)

M1: Establishes a correct method by finding one of the (correct) inequalities

States either  $k > 8$  (condone  $k \geq 8$ ) or  $k < 0$  (condone  $k \leq 0$ )

Condone for this mark  $y \leftrightarrow k$  or  $f(x) \leftrightarrow k$  and  $8 < k < 0$

A1: Fully correct solution in the form  $\{k : k > 8\} \cup \{k : k < 0\}$  or  $\{k | k > 8\} \cup \{k | k < 0\}$  either way around

but condone  $\{k < 0\} \cup \{k > 8\}$ ,  $\{k : k < 0 \cup k > 8\}$ ,  $\{k < 0 \cup k > 8\}$ . It is not necessary to mention  $\mathbb{R}$ , e.g.  $\{k : k \in \mathbb{R}, k > 8\} \cup \{k : k \in \mathbb{R}, k < 0\}$  Look for  $\{ \}$  and  $\cup$

Do not allow solutions not in set notation such as  $k < 0$  or  $k > 8$ .

(c)

M1: Realises that the equation of  $C$  is of the form  $y = ax(x-6)^2$ . Condone with  $a = 1$  for this mark.

So award for sight of  $ax(x-6)^2$  even with  $a = 1$

dM1: Substitutes (2,8) into the form  $y = ax(x-6)^2$  and attempts to find the value for  $a$ .

It is dependent upon having an equation, which the ( $y = \dots$ ) may be implied, of the correct form.

A1: Uses all of the information to form a correct **equation** for  $C$   $y = \frac{1}{4}x(x-6)^2$  o.e.

ISW after a correct answer. Condone  $f(x) = \frac{1}{4}x(x-6)^2$  but not  $C = \frac{1}{4}x(x-6)^2$ .

Allow this to be written down for all 3 marks

## Examples of alternative methods

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**Alternative I part (c):****Using the form  $y = ax^3 + bx^2 + cx$  and setting up then solving simultaneous equations.****There are various versions of this but can be marked similarly**M1: Realises that the equation of  $C$  is of the form  $y = ax^3 + bx^2 + cx$  and forms two equations in  $a$ ,  $b$  and  $c$ . Condone with  $a = 1$  for this mark.Note that the form  $y = ax^3 + bx^2 + cx + d$  is M0 until  $d$  is set equal to 0.

There are four equations that could be formed, only two are necessary for this mark.

Condone slips

Using  $(6, 0) \Rightarrow 216a + 36b + 6c = 0$

Using  $(2, 8) \Rightarrow 8a + 4b + 2c = 8$

Using  $\frac{dy}{dx} = 0$  at  $x = 2 \Rightarrow 12a + 4b + c = 0$

Using  $\frac{dy}{dx} = 0$  at  $x = 6 \Rightarrow 108a + 12b + c = 0$

dM1: Forms and solves three different equations, one of which must be using  $(2, 8)$  to find values for  $a$ ,  $b$  and  $c$ . A calculator can be used to solve the equationsA1: Uses all of the information to form a correct equation for  $C$   $y = \frac{1}{4}x^3 - 3x^2 + 9x$  o.e.

ISW after a correct answer. Condone  $f(x) = \frac{1}{4}x^3 - 3x^2 + 9x$

.....  
**Alternative II part (c)****Using the gradient and integrating**M1: Realises that the gradient of  $C$  is zero at 2 and 6 so sets  $f'(x) = k(x-2)(x-6)$  or **and** attempts to integrate. Condone with  $k = 1$ dM1: Substitutes  $x = 2, y = 8$  into  $f(x) = k(\dots x^3 + \dots x + \dots)$  and finds a value for  $k$ A1: Uses all of the information to form a correct equation for  $C$   $y = \frac{3}{4}\left(\frac{1}{3}x^3 - 4x^2 + 12x\right)$  o.e.

ISW after a correct answer. Condone  $f(x) = \frac{3}{4}\left(\frac{1}{3}x^3 - 4x^2 + 12x\right)$   
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Question	Scheme	Marks	AOs
4 (i)	For setting up the contradiction: There exists integers $p$ and $q$ such that $pq$ is even and both $p$ and $q$ are odd	B1	2.5
	For example, sets $p = 2m + 1$ and $q = 2n + 1$ and then attempts $pq = (2m + 1)(2n + 1) = \dots$	M1	1.1b
	Obtains $pq = (2m + 1)(2n + 1) = 4mn + 2m + 2n + 1$ $= 2(2mn + m + n) + 1$ States that this is odd, giving a contradiction so "if $pq$ is even, then at least one of $p$ and $q$ is even" *	A1*	2.1
		(3)	
(ii)			
	$(x + y)^2 < 9x^2 + y^2 \Rightarrow 2xy < 8x^2$	M1	2.2a
	States that as $x < 0 \Rightarrow 2y > 8x$ $\Rightarrow y > 4x$ *	A1*	2.1
		(2)	
<b>(5 marks)</b>			
<b>Notes:</b>			

(i)

B1: For using the "correct"/ allowable language in setting up the contradiction.

Expect to see a minimum of

- "assume" or "let" or "there is " or other similar words
- " $pq$  is even" and " $p$  and  $q$  are (both) odd"

M1: Uses a correct algebraic form for  $p$  and  $q$  and attempting to multiply.Allow any correct form so  $p = 2n + 1$  and  $q = 2m + 3$  would be fine to use**Different variables must be used** for  $p$  and  $q$ , so  $p = 2n + 1$  and  $q = 2n - 1$  would be M0

A1\*: Full argument .

This requires (1) a correct calculation for their  $pq$ 

(2) a correct reason and conclusion that it is odd

E.g.  $(2m + 1)(2n + 1) = 4mn + 2m + 2n + 1 = 2(2mn + m + n) + 1 = \text{odd}$

E.g.  $(2m - 1)(2n + 1) = 4mn + 2m - 2n - 1 = \text{even} + \text{even} - \text{even} - 1 = \text{odd}$

and (3) a minimal statement implying that they have proven what was required which could be QED, proven etc

Note that B0 M1 A1 is possible

(ii)

M1: For multiplying out and cancelling terms before proceeding to a correct intermediate line such as

$$2xy < 8x^2 \text{ o.e. such as } 2x(4x - y) > 0$$

A1\*: Full and rigorous proof with reason shown as to why inequality reverses. The point at which it reverses must be correct and a correct reason given

See scheme

Alt:  $2xy < 8x^2 \Rightarrow xy - 4x^2 < 0 \Rightarrow x(y - 4x) < 0$

as  $x < 0$ ,  $(y - 4x) > 0 \Rightarrow y > 4x$  scores M1 A1

So, the following should be scored M1 A0 as line 3 is incorrect

$$2xy - 8x^2 < 0$$

$$\Rightarrow 2xy < 8x^2$$

$$\Rightarrow y < 4x$$

$$\Rightarrow y > 4x \text{ as } x < 0$$

There should be no incorrect lines in their proof